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MATHEMATICAL MODEL OF THE PROCESSES OF FLOW OF A LUBRICATING LAYER AND ELASTOPLASTIC DEFORMATION OF A PRODUCT

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An original mathematical model for description of the combined flow of a viscous lubricating layer and elastoplastic drawing deformation of a multilayer product has been developed. The theory of plastic flow with linear anisotropic hardening has been used for description of the material's behavior. The O. M. Belotserkovskii approach has been applied to the determination of the pressure in the lubricating layer. The problem of elastoplastic deformation of the product in the hydrodynamic-friction regime has been solved using the model developed.

Introduction. The technology of deformation of products in the hydrodynamic-friction regime is widely used in industry; it implies that the tool and the product deformed are separated by a thin lubricating layer. In particular, bimetallic products from powder-copper-based dispersion-hardened composite materials are manufactured according to this technology. It is expected that the above technology will be used in the production of superconducting cables.

The characteristics of the state of products in deformation in the hydrodynamic-friction regime are investigated based on the methods of the mechanics of a deformable rigid body and on hydrodynamics methods. The problems of analysis of the elastoplastic deformation of metals have been considered in [1–10]. The methods of solution of Navier–Stokes equations have been studied in [11–14]. The problems of stability of flow of a thin lubricating layer in plastic metal working have been investigated in [15–18].

Different models [7, 19–26] have been developed for correct description of the deformation in the hydrodynamic-friction regime. However, in these models, the problem is subdivided into two independent ones: the problem of deformation of a product and that of flow of a lubricating layer. Therefore, it is of scientific and practical interest to construct a mathematic model free of the above drawback.

Formulation of the Problem. The method of hydrodynamic injection of a lubricant [7, 22] involves the production of a high pressure in it due to the hydrodynamic effect appearing in friction of the lubricant against a moving bar (Fig. 1). Lubricant 4, which is the free state in vessel 3, is captured by a moving bar 1 and is entrained in microgap 2 between the head tube and the bar. As a result the pressure of the lubricant near the deformation zone increases to a value ensuring its injection into the contact region.

It is assumed that the deformation of the bar is nonstationary, nonisothermal, and axisymmetric; the plastic-deformation energy completely dissipates to heat; the lubricant is thought to be viscous and incompressible; the bar consists of isotropic materials differing in properties and with the initially known boundary. The theory of plastic flow with linear anisotropic hardening is applied to the description of the behavior of the materials.

Let the multilayer product and the lubricating layer occupy, at a certain instant of time $t \in [0, t_1]$, a bounded region $\Omega = \Omega_{ep}^1 \cup \Omega_{ep}^2 \cup \Omega_{liq} \subset R^3$ with boundary Γ and boundaries of the materials Γ_c^1 between the layers Ω_{ep}^1 and Ω_{ep}^2 of the product and Γ_c^2 between Ω_{ep}^1 and the lubricating layer Ω_{liq}. The closure of the region Ω_{liq} with boundary Γ_{liq} is determined as $\bar{\Omega}_{liq} = \Omega_{liq} \cup \Gamma_{liq}$. We denote the region of elastoplastic deformation with boundary Γ_{ep}, $\bar{\Omega}_{ep} = \Omega_{ep} \cup \Gamma_{ep}$, by $\Omega_{ep} = \Omega_{ep}^1 \cup \Omega_{ep}^2$.

The stressed-deformed state of the multilayer product and flow of the lubricant are described by the general system of the equations of motion

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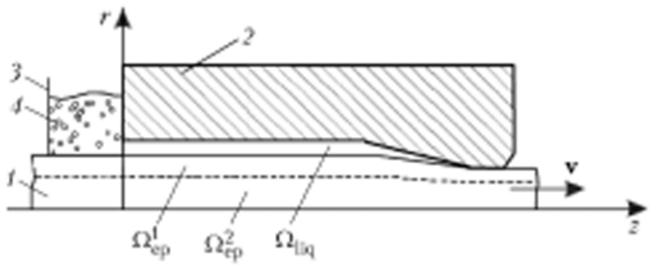

Fig. 1. Diagram of elastoplastic deformation with the hydrodynamic injection of a lubricant.

$$\rho \frac{d\mathbf{v}}{dt} = \boldsymbol{\nabla}\cdot\sigma + \rho \mathbf{F}, \quad \mathbf{x} \in \Omega, \quad t \in [0, t_1], \tag{1}$$

heat conduction

$$\frac{d(\rho c T)}{dt} = \boldsymbol{\nabla}\cdot(\chi \boldsymbol{\nabla} T) + W, \quad \mathbf{x} \in \Omega, \quad t \in [0, t_1], \tag{2}$$

$$W = \begin{cases} \sigma_i \dot{\varepsilon}_i^p, & \mathbf{x} \in \Omega_{ep}, \\ \eta\, (\boldsymbol{\nabla}\mathbf{v} + (\boldsymbol{\nabla}\mathbf{v})^t)\, \boldsymbol{\nabla}\mathbf{v}, & \mathbf{x} \in \Omega_{liq}, \end{cases} \tag{3}$$

and continuity

$$\boldsymbol{\nabla}\cdot(\rho \mathbf{v}) = 0, \quad \mathbf{x} \in \Omega_{liq}, \tag{4}$$

by the dependence of the displacement **u** on the velocity **v**

$$\frac{d\mathbf{u}}{dt} = \mathbf{v}, \quad \mathbf{x} \in \Omega_{ep}, \quad t \in [0, t_1], \tag{5}$$

by the physical and geometric relations

$$d\sigma = f_\sigma(d\varepsilon, dT), \quad d\varepsilon = \frac{1}{2}\left(\boldsymbol{\nabla}\mathbf{v} + (\boldsymbol{\nabla}\mathbf{v})^t\right) dt, \quad \mathbf{x} \in \overline{\Omega}_{ep}, \tag{6}$$

$$\sigma = f_\sigma(\dot{\varepsilon}), \quad \dot{\varepsilon} = \frac{1}{2}\left(\boldsymbol{\nabla}\mathbf{v} + (\boldsymbol{\nabla}\mathbf{v})^t\right), \quad \mathbf{x} \in \overline{\Omega}_{liq}, \tag{7}$$

by the initial conditions ($t = 0$)

$$\mathbf{v} = \mathbf{v}_0, \quad T = T_0, \quad \mathbf{x} \in \overline{\Omega}, \tag{8}$$

$$\mathbf{u} = \mathbf{u}_0, \quad \varepsilon = \varepsilon_0, \quad \sigma = \sigma_0, \quad \mathbf{x} \in \overline{\Omega}_{ep}, \tag{9}$$

$$P = P_0, \quad \mathbf{x} \in \overline{\Omega}_{liq}, \tag{10}$$

by the boundary force

$$\sigma\cdot\mathbf{n}=F_\Gamma\,,\quad \mathbf{x}\in\Gamma_F\,, \tag{11}$$

kinematic

$$\mathbf{v}=\mathbf{v}_\Gamma\,,\quad \mathbf{x}\in\Gamma_v\,, \tag{12}$$

and thermal conditions

$$T=T_\Gamma\,,\quad \mathbf{x}\in\Gamma_T\,, \tag{13}$$

$$\chi\boldsymbol{\nabla}T\cdot\mathbf{n}=-\,\alpha\,(T-T_\Gamma)+\frac{W_\tau}{2}\,,\quad \mathbf{x}\in\Gamma_{\mathrm{h.f}}\,, \tag{14}$$

$$W_\tau=\begin{cases}|\,\mathbf{F}_\tau\cdot\mathbf{v}\,|\,, & \mathbf{x}\in\Gamma_\tau\,,\\ 0\,; & \mathbf{x}\notin\Gamma_\tau\,,\end{cases}$$

and by the conditions at the boundaries of the materials

$$\mathbf{v}^1=\mathbf{v}^2\,,\quad \mathbf{F}^1+\mathbf{F}^2=0\,,\quad \chi^1\boldsymbol{\nabla}T^1\cdot\mathbf{n}^1+\chi^2\boldsymbol{\nabla}T^2\cdot\mathbf{n}^2=0\,,\quad T^1=T^2\,,\quad \mathbf{x}\in\Gamma_{\mathrm{c}}\,. \tag{15}$$

It is necessary to find the functions $\mathbf{v}(\mathbf{x}, t)$, $T(\mathbf{x}, t)$, $\mathbf{u}(\mathbf{x}, t)$, $\varepsilon(\mathbf{x}, t)$, $\sigma(\mathbf{x}, t)$, and $P(\mathbf{x}, t)$ satisfying system (1)–(15) and to determine the contact and free boundaries and the boundaries of the materials.

Resolving Relations. According to [3, 27], the relationship between the increments in the stress and deformation tensors for the elastoplastic material is taken in the form

$$d\sigma=D\cdot\cdot d\varepsilon+RdT\,. \tag{16}$$

For the lubricant, the dependence of the stress tensor on the velocity vector has the form [13]

$$\sigma=-\,\delta P+\eta\left(\boldsymbol{\nabla}\mathbf{v}+(\boldsymbol{\nabla}\mathbf{v})^{\mathrm{t}}\right). \tag{17}$$

In constructing resolving relations, we use the Galerkin method with a finite-element approximation of the solution. A set of trial vector functions Φ_i, $i = 1, 2, ...,$ is constructed based on a complete and closed system of scalar functions φ_i, $i = 1, 2, ...,$ in the form

$$\boldsymbol{\Phi}_1=\{\varphi_1, 0, 0\}\,,\quad \boldsymbol{\Phi}_2=\{0, \varphi_1, 0\}\,,\quad \boldsymbol{\Phi}_3=\{0, 0, \varphi_1\}\,,\quad \boldsymbol{\Phi}_4=\{\varphi_2, 0, 0\}\,,\quad \boldsymbol{\Phi}_5=\{0, \varphi_2, 0\}\,, ... \tag{18}$$

The set of vector functions (18) forms a complete and closed system. We assume that the scalar product is $(\mathbf{Y}, \boldsymbol{\Phi}_k) = \int_\Omega \mathbf{Y}\cdot\boldsymbol{\Phi}_k d\Omega = 0$ for all $k = 1, 2, ...,$ and $\mathbf{Y} = \mathbf{Y}(y_1, y_2, y_3)$. We show that this is possible only when $\mathbf{Y} = 0$. We have

$$(\mathbf{Y}, \Phi_1)=\int_\Omega \mathbf{Y}\cdot\Phi_1 d\Omega=\int_\Omega y_1\Phi_1^1+y_2\Phi_1^2+y_3\Phi_1^3 d\Omega=\int_\Omega y_1\varphi_1 d\Omega$$

for $k = 1$,

$$(\mathbf{Y}, \Phi_2)=\int_\Omega \mathbf{Y}\cdot\boldsymbol{\Phi}_2 d\Omega=\int_\Omega y_2\varphi_1 d\Omega$$

for $k = 2$,

$$(\mathbf{Y}, \Phi_3) = \int_\Omega \mathbf{Y}\cdot\mathbf{\Phi}_3 d\Omega = \int_\Omega y_3 \varphi_1 d\Omega$$

for $k = 3$,

$$(\mathbf{Y}, \Phi_4) = \int_\Omega \mathbf{Y}\cdot\mathbf{\Phi}_4 d\Omega = \int_\Omega y_1 \varphi_2 d\Omega$$

for $k = 4$, and

$$(\mathbf{Y}, \Phi_5) = \int_\Omega \mathbf{Y}\cdot\mathbf{\Phi}_5 d\Omega = \int_\Omega y_2 \varphi_2 d\Omega$$

for $k = 5$. Generalizing the resulting expressions, we write three expressions which enable us to determine the scalar product for any k:

$$(\mathbf{Y}, \mathbf{\Phi}_k) = \int_\Omega \mathbf{Y}\cdot\mathbf{\Phi}_k d\Omega = \int_\Omega y_1 \Phi_k^1 d\Omega = \int_\Omega y_1 \varphi_{\frac{k+2}{3}} d\Omega\,, \quad k = 1, 4, 7, \dots\,, \tag{19}$$

$$(\mathbf{Y}, \mathbf{\Phi}_k) = \int_\Omega \mathbf{Y}\cdot\mathbf{\Phi}_k d\Omega = \int_\Omega y_2 \Phi_k^2 d\Omega = \int_\Omega y_2 \varphi_{\frac{k+1}{3}} d\Omega\,, \quad k = 2, 5, 8, \dots\,, \tag{20}$$

$$(\mathbf{Y}, \mathbf{\Phi}_k) = \int_\Omega \mathbf{Y}\cdot\mathbf{\Phi}_k d\Omega = \int_\Omega y_3 \Phi_k^3 d\Omega = \int_\Omega y_3 \varphi_{\frac{k}{3}} d\Omega\,, \quad k = 3, 6, 9, \dots\,. \tag{21}$$

By virtue of the completeness of the functions φ_i, $k = 1, 2, \dots$, expressions (19)–(21) vanish when $y_1 = 0$, $y_2 = 0$, and $y_3 = 0$, i.e., $\mathbf{Y} = 0$. Consequently, the set of vector functions (18) forms a complete system.

In view of the closeness of the system of functions φ_k, $k = 1, 2, \dots$, there exist expansion coefficients a_{3k-2}, a_{3k-1}, and a_{3k}, $k = 1, 2, \dots$, such that the inequalities

$$\left\| y_1 - \sum_{k=1}^{\infty} a_{3k-2}\varphi_k \right\| \le \xi\,, \quad \left\| y_2 - \sum_{k=1}^{\infty} a_{3k-1}\varphi_k \right\| \le \xi\,, \quad \left\| y_3 - \sum_{k=1}^{\infty} a_{3k}\varphi_k \right\| \le \xi \tag{22}$$

hold. Based on (22), we perform the evaluation

$$\left\| \mathbf{Y} - \sum_{k=1}^{\infty} a_k \mathbf{\Phi}_k \right\|^2 = \left\| y_1 - \sum_{k=1}^{\infty} a_k \Phi_k^1 \right\|^2 + \left\| y_2 - \sum_{k=1}^{\infty} a_k \Phi_k^2 \right\|^2 + \left\| y_3 - \sum_{k=1}^{\infty} a_k \Phi_k^3 \right\|^2$$

$$= \left\| y_1 - \sum_{k=1}^{\infty} a_{3k-2}\varphi_k \right\|^2 + \left\| y_2 - \sum_{k=1}^{\infty} a_{3k-1}\varphi_k \right\|^2 + \left\| y_3 - \sum_{k=1}^{\infty} a_{3k}\varphi_k \right\|^2 \le \xi^2 + \xi^2 + \xi^2\,.$$

Consequently, we obtain

$$\left\| \mathbf{Y} - \sum_{k=1}^{\infty} a_k \mathbf{\Phi}_k \right\| \le \sqrt{3}\,\xi\,, \tag{23}$$

i.e., the set of vector functions (18) forms a closed system.

For the elastoplastic material, the equations of motion (1), with account for the physical and geometric relations for the finite element numbered k, are written in the form of the system of equations

$$-\sum_{i=1}^{m} \int_{\Omega_{\mathrm{ep}}} \left([B_k]\,[D^{\mathrm{ep}}]\,[B_i]^{\mathrm{t}} + \rho\,[\varphi_k]\,\big(\{\varphi_i\}\,\boldsymbol{\nabla}\big)\,\mathbf{v}^{n-1} \right) d\Omega\,\left\{\mathbf{v}_i^n\right\} \int_{\Omega_{\mathrm{ep}}} \left([B_k]\,\{R\}\,dT \right) d\Omega$$

$$-\sum_{i=1}^{m} \int_{\Omega_{\mathrm{ep}}} \rho\,[\varphi_k]\,\{\varphi_i\}\,d\Omega\,\frac{\partial \left\{\mathbf{v}_i^n\right\}}{\partial t} = -\int_{\Gamma_{\mathrm{ep}}} [\varphi_k]\,\{\mathbf{F}\}\,d\Gamma\,, \tag{24}$$

where $\mathbf{v}^{n-1}$ is the velocity value determined on the previous time step and $\mathbf{v}^n$ is the sought velocity. For the region with a lubricant, the equations of motion (1), with account for (17), are transformed as

$$-\sum_{i=1}^{m} \int_{\Omega_{\mathrm{liq}}} \left([B_k]\,[D^{\mathrm{liq}}]\,[B_i]^{\mathrm{t}} + \rho\,[\varphi_k]\,\big(\{\varphi_i\}\,\boldsymbol{\nabla}\big)\,\mathbf{v}^{n-1} \right) d\Omega\,\left\{\mathbf{v}_i^{n-1/2}\right\} - \int_{\Omega_{\mathrm{liq}}} [B_k]\,P\,\{\delta\}\,d\Omega$$

$$-\sum_{i=1}^{m} \int_{\Omega_{\mathrm{liq}}} \rho\,[\varphi_k]\,\{\varphi_i\}\,d\Omega\,\frac{\partial \left\{\mathbf{v}_i^{n-1/2}\right\}}{\partial t} = -\int_{\Gamma_{\mathrm{liq}}} [\varphi_k]\,\{\mathbf{F}\}\,d\Gamma\,, \tag{25}$$

where $\mathbf{v}^{n-1/2}$ is the sought intermediate velocity whose value is to be subsequently refined. The form of the matrices of velocity gradients $[B]$ and physical properties $[D^{\mathrm{ep}}]$ and $[D^{\mathrm{liq}}]$ has been given in [28]. To refine the velocity field and to determine pressure in the liquid lubricant we use the O. M. Belotserkovskii approach [11], according to which we introduce the additional pressure δP determined as the solution of the differential equation

$$\Delta\,(\delta P) = \frac{\rho}{\tau}\,\nabla\cdot\mathbf{v}^{n-1/2}\,, \quad \mathbf{x} \in \Omega_{\mathrm{liq}}\,.$$

The distribution δP found enables us to refine the pressure field

$$P^n = P^{n-1} + \delta P\,, \quad \mathbf{x} \in \Omega_{\mathrm{liq}}\,,$$

and the components of the velocity vector

$$\mathbf{v}^n = \mathbf{v}^{n-1/2} - \frac{\tau}{\rho}\,\nabla\,(\delta P)\,, \quad \mathbf{x} \in \Omega_{\mathrm{liq}}\,,$$

satisfying the continuity equation in this case. The resolving relation for determination of a correction to pressure for the finite element numbered k is written in the form

$$\sum_{i=1}^{m} \int_{\Omega_{\mathrm{liq}}} \boldsymbol{\nabla}\varphi_i \cdot \boldsymbol{\nabla}\varphi_k d\Omega\,(\delta P_i) = \sum_{i=1}^{m} \int_{\Gamma_{\mathrm{liq}}} (\nabla\varphi_i \cdot \mathbf{n})\,\varphi_k d\Gamma\,(\delta P_i) - \int_{\Omega_{\mathrm{liq}}} \frac{\rho}{\tau}\,\nabla\mathbf{v}^{n-1/2}\varphi_k d\Omega\,. \tag{26}$$

The resolving relation for determination of temperature for the finite element numbered k has the form

$$\sum_{i=1}^{m} \int_{\Omega} \rho c \varphi_i \varphi_k d\Omega \frac{\partial T_i}{\partial t} + \sum_{i=1}^{m} \int_{\Omega} \left(\rho c \, (\mathbf{v}^n \cdot \nabla \varphi_i) \, \varphi_k + \chi \nabla \varphi_i \cdot \nabla \varphi_k \right) d\Omega T_i$$

$$+ \sum_{i=1}^{m} \int_{\Gamma} \alpha \varphi_i \varphi_k d\Gamma T_i = \int_{\Gamma} \alpha T_{\Gamma} \varphi_k + \frac{W_{\tau}}{2} \varphi_k d\Gamma + \int_{\Omega} W \varphi_k d\Omega \, . \tag{27}$$

Equations (24), (25), and (27) are nonstationary. To solve them we use the Crank–Nicolson difference scheme [29], according to which we employ, for determination of the unknown function ω from a nonstationary equation of the parabolic type,

$$\frac{\partial \omega}{\partial t} + A\omega = f \tag{28}$$

with the initial condition

$$\omega \, (0) = g \tag{29}$$

the corresponding difference equation

$$\frac{\omega^n - \omega^{n-1}}{\tau} + \Lambda^{n-1} \frac{\omega^n + \omega^{n-1}}{2} = f^{n-1} \, , \quad \omega^0 = g \, , \tag{30}$$

where $\Lambda^{n-1} = \frac{1}{2}(A^n + A^{n-1})$ and $f^{(n-1)} = f(t^{n-1/2})$. The employment of the difference scheme has enabled us to pass from the initially nonlinear problem to a linearized one.

Algorithm of Solution. For simultaneous solution of problem (5)–(7) and (24)–(27) with boundary conditions (8)–(15) we use the following numerical algorithm. Let the stressed-deformed state of the material be known for an arbitrary instant of time t. Then the components of the velocity vector $\mathbf{v}$ for $t^{n-1} + \Delta t$ are determined for the entire region under study, when the resolving relations (24) and (25) are simultaneously used. The increments of the displacement-vector components and the deformation and stress tensors in the metal are determined from the known velocity field (5)–(7); from Eq. (26), the increment in the pressure δP is found for the liquid and the components of the velocity and pressure fields are corrected; the strength of internal heat sources is computed. The field of temperature T is calculated from relation (27) with allowance for the surface and internal heat sources. Next, elastic and plastic zones are determined for the metal. The algorithm of separation has been given in [28]. This enables us to determine the components of the tensors D and R for the next step of calculations. From the increment in the displacement vector $\mathbf{u}$ found, we compute the position of the product's boundaries in space and refine new free and contact boundaries and the boundaries of the materials. To determine the boundary with friction we compute the elements on the generatrix of the tool. Then we pass to the next computational step. This continues until the required instant of time is reached. Thus, the algorithm enables us to track the evolution of the stressed-deformed state of the product and the velocity and pressure fields in the lubricating layer.

Verification. The mathematical model has been verified on the following problems: determination of the stressed-deformed state of a long cylinder under thermoelastic and plastic deformation (the maximum error between the numerical and exact solutions [30, 31] amounted to 1.88% in elastic deformation, to 0.21% in thermoelastic deformation, and to 6.28% in plastic deformation) and determination of the velocity and pressure vector in the case of flow of a liquid in a cylindrical channel (the maximum error between the numerical and exact solutions [13] amounted to 1.84%). The use of the quadratic functions

$$\psi_i = \varphi_i \, (2\varphi_i - 1) \, , \quad \psi_j = \varphi_j \, (2\varphi_j - 1) \, , \quad \psi_k = \varphi_k \, (2\varphi_k - 1) \, ,$$

$$\psi_l = 4\varphi_i \varphi_j \, , \quad \psi_m = 4\varphi_k \varphi_k \, , \quad \psi_n = 4\varphi_k \varphi_i \, ,$$

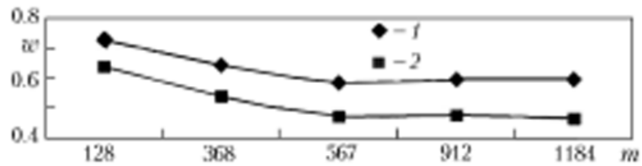

Fig. 2. Convergence of the solution of the continuity equation: 1) linear trial functions; 2) quadratic trial functions.

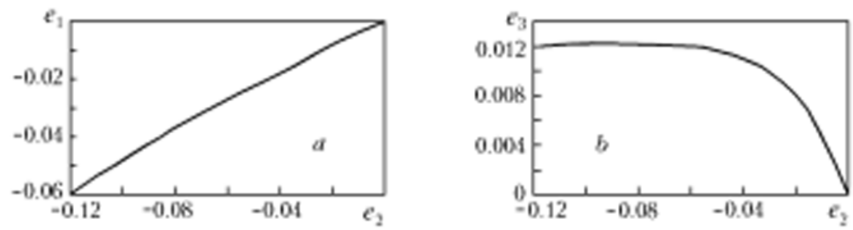

Fig. 3. Projections of the deformation trajectories onto the plane e_1, e_2 (a) and e_3, e_2 (b).

constructed based on the functions of first order

$$\varphi_i\,(r, z) = \alpha_i + \beta_i z + \gamma_i r\ ,\quad \varphi_j\,(r, z) = \alpha_j + \beta_j z + \gamma_j r\ ,\quad \varphi_k\,(r, z) = \alpha_k + \beta_k z + \gamma_k r\ ,$$

in approximation of the pressure enables us to obtain a more exact solution of the problem on the compound motion of a liquid. The exactness ω and convergence of the solution of the continuity equation as a function of the order of approximation and the number of finite elements m is plotted in Fig. 2. As a measure of error of the solution of the continuity equation, we use the integral norm

$$\| \nabla\cdot\mathbf{v} \| = \sqrt{\frac{1}{S}\int\limits_{\Omega_{\text{liq}}} (\nabla\cdot\mathbf{v})^2 d\Omega}\ ,$$

where $S = \int\limits_{\Omega_{\text{liq}}} d\Omega$. For the exact solution, we have $\|\nabla\cdot\mathbf{v}\| = 0$.

To evaluate the applicability of plastic-flow theory we have investigated the problem of determination of the stressed-deformed state of a bar under deformation in the mixed-friction regime without allowance for the lubricating layer; we constructed, at different instants of time, the trajectories of deformation of material particles in the A. A. Il'yushin space

$$e_1 = e_{rr}\sqrt{\frac{3}{2}}\ ,\quad e_2 = \sqrt{2}\left(\frac{1}{2}\,e_{rr} + e_{\varphi\varphi}\right),\quad e_3 = e_{r\varphi}\sqrt{2}\ ,\quad e_4 = e_{\varphi z}\sqrt{2}\ ,\quad e_5 = e_{zr}\sqrt{2}$$

and checked the fulfillment of the inequality

$$\zeta < \kappa^{-1}\ .$$

The deformation trajectories satisfy the relations of small curvature (Fig. 3).

Results. Using the mathematical model developed, we solve the joint problem of flow of a lubricating layer and deformation of a product in the hydrodynamic-friction regime (Fig. 4).

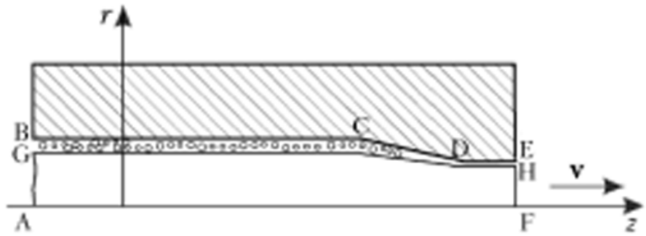

Fig. 4. Deformation of the product in the hydrodynamic-friction regime.

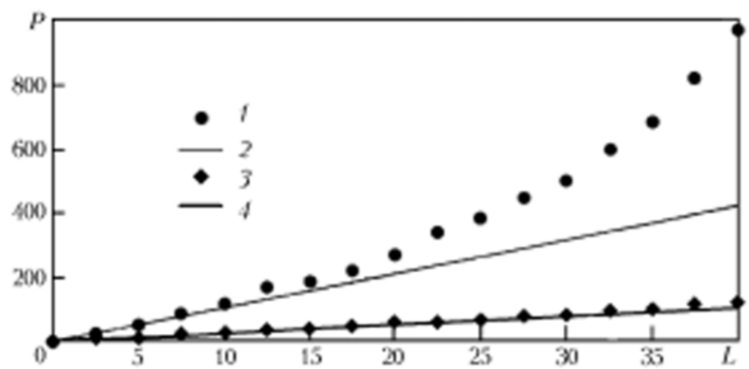

Fig. 5. Distribution of pressure P in the lubricating layer vs. length of the head tube L and thickness of the lubricant layer h: 1 and 2) numerical and analytical solutions for h = 0.05 mm; 3 and 4) numerical and analytical solutions for h = 0.1 mm. P, mPa; L, mm.

The practice of deformation in the hydrodynamic-friction regime involves the use of pressure head tubes producing the high pressure of a lubricant in the deformation zone. The hydrodynamic-friction friction regime is attained owing to the effect of injection of the lubricant into the gap between the billet's surface and the pressure head tube by a moving product. The dependence of the pressure on the length of the heat tube L and the thickness of the lubricant layer h has been investigated in [28]. The solutions were obtained numerically and analytically. The Barus law was used in numerical calculations for description of the pressure dependence of the lubricant viscosity. Figure 5 shows the parameters of the head tube that ensure a high pressure in the lubricant before the entry into the deformation zone.

It is taken that the axial component v_1 of the vector of velocity $\mathbf{v}$ of motion of the bar at exit from the drawing die is equal to 5 m/sec, the radial component is absent, the die half-angle is $\theta = 6^{o}$, and the drawing coefficient is $\lambda = 1.2$. The initial radius is equal to 2.94 mm, the length of the head tube is 30 mm, and the thickness of the lubricant layer in the gap is h = 0.05 mm. We take MS-20 mineral oil as the lubricant.

In the problem, we adopt the following initial and boundary conditions (Fig. 4):

$$\mathbf{u} = \mathbf{u}_0\,,\ \ \varepsilon = \varepsilon_0\,,\ \ \sigma = \sigma_0\,,\ \ \mathbf{v} = \mathbf{v}_0\,,\ \ T = T_0\,,\ \ P = P_0\,,\ \ F_\tau|_{\mathrm{AG\cup GB}} = 0\,,\ \ F_\mathrm{n}|_{\mathrm{AG\cup GB}} = 0\,,$$

$$T|_{\mathrm{AG\cup GB}} = T_1\,,\ \ v_r|_{\mathrm{BC}} = 0\,,\ \ v_z|_{\mathrm{BC}} = 0\,,\ \ \chi\boldsymbol{\nabla}T|_{\mathrm{BC}}\cdot\mathbf{n} = -\,\alpha\,(T|_{\mathrm{BC}} - T_\Gamma)\,,\ \ v_r|_{\mathrm{CD}} = 0\,,\ \ v_z|_{\mathrm{CD}} = 0\,,$$

$$\chi\nabla T|_{\mathrm{CD}}\cdot\mathbf{n} = -\,\alpha\,(T|_{\mathrm{CD}} - T_\Gamma)\,,\ \ v_r|_{\mathrm{DE}} = 0\,,\ \ v_z|_{\mathrm{DE}} = 0\,,\ \ \chi\boldsymbol{\nabla}T|_{\mathrm{DE}}\cdot\mathbf{n} = -\,\alpha\,(T|_{\mathrm{DE}} - T_\Gamma)\,,$$

$$\left.\frac{\partial v_r}{\partial \mathbf{n}}\right|_{\mathrm{EH}} = 0\,,\ \ \left.\frac{\partial v_z}{\partial \mathbf{n}}\right|_{\mathrm{EH}} = 0\,,\ \ \chi\boldsymbol{\nabla}T|_{\mathrm{EH}}\cdot\mathbf{n} = 0\,,\ \ F_\tau|_{\mathrm{HF}} = 0\,,\ \ v_z|_{\mathrm{HF}} = v_1\,,\ \ \chi\boldsymbol{\nabla}T|_{\mathrm{HF}}\cdot\mathbf{n} = 0\,,$$

$$F_\tau|_{\mathrm{FA}} = 0\,,\ \ v_r|_{\mathrm{FA}} = 0\,,\ \ \chi\boldsymbol{\nabla}T|_{\mathrm{FA}}\cdot\mathbf{n} = 0\,,\ \ \delta P|_{\mathrm{GB}} = 0\,,$$

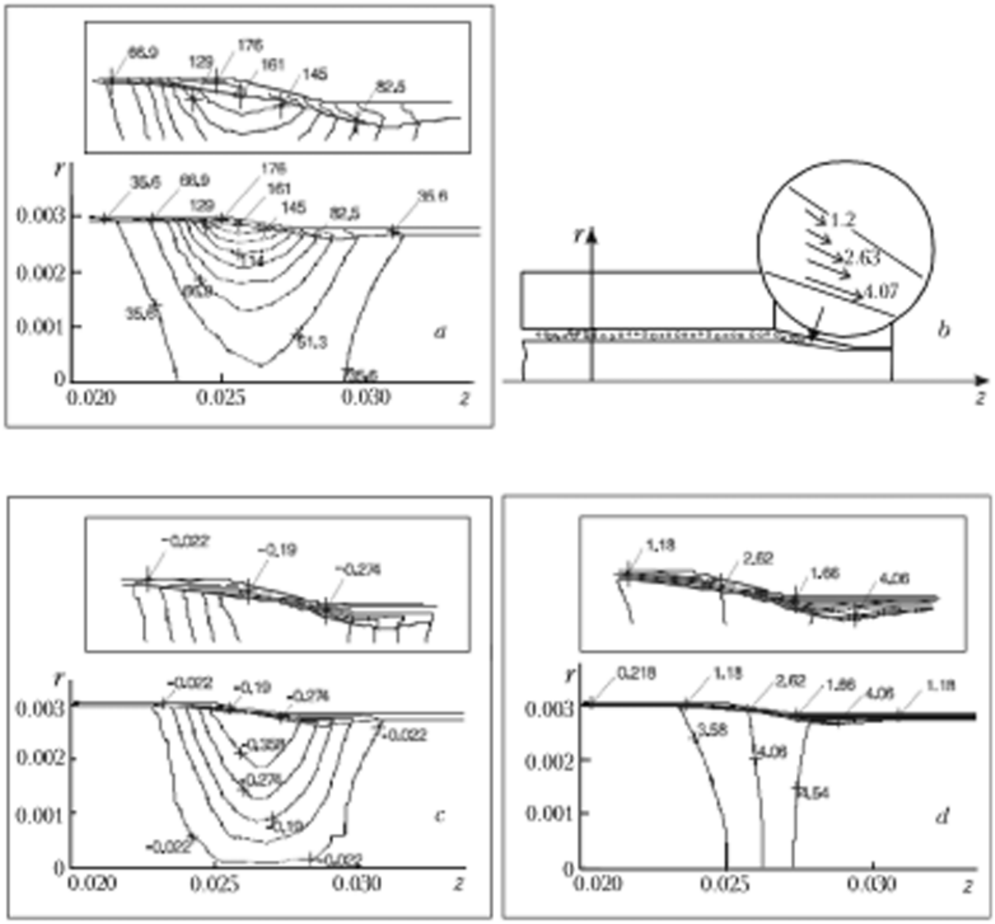

Fig. 6. Characteristics of deformation of the product in the hydrodynamic-friction regime for the distribution of the: a) temperature T, b) velocity in the lubricating layer $\mathbf{v}$, c) radial velocity v_r, and axial velocity v_z. r and z, m.

$$\left.\frac{\partial \delta P}{\partial \mathbf{n}}\right|_{BC} = \left.\frac{\partial \delta P}{\partial \mathbf{n}}\right|_{CD} = \left.\frac{\partial \delta P}{\partial \mathbf{n}}\right|_{DE} = \left.\frac{\partial \delta P}{\partial \mathbf{n}}\right|_{EH} = \left.\frac{\partial \delta P}{\partial \mathbf{n}}\right|_{GH} = 0\,.$$

Conditions at the boundary between the metal and the lubricating layer are specified in the form

$$\mathbf{v}^1\,|_{GH} = \mathbf{v}^2\,|_{GH}\,, \quad F_\tau^1\,|_{GH} + F_\tau^2\,|_{GH} = 0\,, \quad F_n^1\,|_{GH} + F_n^2\,|_{GH} = 0\,, \quad T^1\,|_{GH} = T^2\,|_{GH}\,,$$

$$\chi^1 \nabla T^1 \cdot \mathbf{n}\,|_{GH} + \chi^2 \nabla T^2 \cdot \mathbf{n}\,|_{GH} = 0\,.$$

Heating in the lubricating layer is by viscous-friction forces (Fig. 6a). The product's surface is heated owing to contact heat exchange and plastic-deformation energy. Certain experimental data on the temperatures of the product's contact layer under deformation in the hydrodynamic-friction regime are indicated in [7]. In particular, in the drawing of a wire with an initial radius of 2.68 mm, a drawing coefficient $\lambda = 1.16$, and an axial component of the velocity vector $v_1 = 4.75$ m/sec, the heating of the product's surface attains 180°C. The contact-layer temperature obtained with the constructed model is 160 to 176°C, i.e., the deviation from the experimental value does not exceed 11%.

The distribution of the velocity vector and the isolines of its components (Fig. 6b–d) point to the fact that flow resembling Couette flow appears in the lubricating layer. As a rule, it is formed in channels with one moving boundary.

The geometry of the channel influences the appearance of radial velocity-vector components (Fig. 6c). The high pressure in the lubricant contributes to the deformation of the metal and to the change in the product's shape and in the profile of the gap between the tool and the product.

Conclusions. We have mathematically formulated the joint nonstationary nonisothermal axisymmetric boundary-value problem of flow of a lubricating layer and deformation of a multilayer product. Using the Galerkin method we have constructed the resolving relations for partial differential equations describing the deformation of the multilayer product and flow of the lubricating layer. The mathematical model has been verified on problems of the mechanics of a deformable rigid body and fluid mechanics. The results obtained demonstrate the possibility of solving the joint problem of flow of a lubricating layer and elastoplastic deformation of a metallic product and applying the constructed mathematical model to the investigation of the influence of different factors (velocity, geometry of the tool, types of materials and lubricants) on the process in question.

NOTATION

a_k, expansion factors; A, certain operator; B, velocity-gradient matrix, 1/sec; c, specific heat, J/(kg·deg); D, tensor of elastoplastic properties [3]; D^{ep}, matrix of elastoplastic properties in the region Ω_{ep}; D^{liq}, matrix of elastoplastic properties in the region Ω_{liq}; e_{ij}, components of the deviator of the deformation tensor; f, right-hand side of an equation of the parabolic type; f_σ, function prescribing physical relations between the tensors of stresses and deformations or deformation rate depending on the type of region; $\mathbf{F}$, vector of external forces, N; $\mathbf{F}_\tau$, vector of friction force at the boundary, N; $\mathbf{F}_\Gamma$, boundary value of the external-force vector, N; g, initial value of the function ω; h, thickness of the lubricant layer in the gap, mm; L, length of the head tube, mm; m, number of simplexes; $\mathbf{n}$, unit vector of the external normal; P, pressure, Pa; P_0, initial value of pressure, Pa; δP, additional pressure Pa [11]; r, radial coordinate; R, tensor of temperature properties [3]; S, volume of the region with a lubricant; t, time, sec; t_1, time of completion of deformation, sec; T, temperature, $^{\mathrm{o}}$C; T_0, initial value of temperature, $^{\mathrm{o}}$C; T_1, fixed value of temperature, $^{\mathrm{o}}$C; T_Γ, boundary value of temperature, $^{\mathrm{o}}$C; $\mathbf{u}$, displacement vector, m; $\mathbf{u}_0$, initial value of the displacement vector, m; $\mathbf{v}$, velocity vector, m/sec; $\mathbf{v}_0$, initial value of the velocity vector, m/sec; $\mathbf{v}_\Gamma$, boundary value of the velocity vector, m/sec; w, error; W, strength of integral heat sources, W; W_τ, friction power at the boundary, W; $\mathbf{Y}(y_1, y_2, y_3)$, auxiliary vector; z, axial coordinate; $e_{1,2,3,4,5}$, elements of the basis vector in the Il'yushin space; α, thermal expansion coefficient, 1/deg; α_i, β_i, and γ_i, expansion factors; δ, Kronecker symbol; ε, deformation tensor; $\dot{\varepsilon}$, deformation-rate tensor, 1/sec; $\dot{\varepsilon}^{\mathrm{p}}$, intensity of the plastic-deformation rate, 1/sec; ε_0, initial value of the deformation tensor; ζ, radius of curvature of the deformation trajectory; η, coefficient of kinematic viscosity, kg/(m·sec); θ, die half-angle, deg; κ, lag trace; λ, drawing coefficient; Λ, difference operator corresponding to the operator A; ξ, small quantity; ρ, density, kg/m^3; σ, stress tensor, Pa; σ_{i}, stress intensity, Pa; σ_0, initial value of the stress tensor, Pa; τ, artificial time, sec [11]; φ_i, scalar functions forming the complete and closed system; Φ_i, set of trial vector functions; χ, thermal conductivity, W/(m·deg); ψ_i, scalar functions forming the complete and closed system; ω, certain unknown function of one argument. Subscripts and superscripts: c, contact; ep, elastoplastic; i, intensity; i, j, and k, superscripts taking on successive values; liq, liquid; n, normal, r, radial; h.f, heat flux; τ, friction; φ, circular, circumferential; 0, initial; p, plastic; t, transposed.

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